TCHo: a hardware-oriented trapdoor cipher

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ASYMMETRIC ENCRYPTION

There are "security proofs" for public-key encryption: reductions to integer factorization, discrete log, lattice problems, etc.

But. . .

1) on quantum computers, RSA, ECC, ElGamal, etc. are broken
2) on hardware, slow and difficult to implement

On the other hand, LFSR-based stream ciphers fit well lightweight environments.

TCHo

- encrypts with only a LFSR and pseudorandom bits
- decrypts with simple linear algebra over GF(2)
- is semantically secure
- is not known to be harmed by quantum computers

TCHo AND RSA

Public key:

- **TCHo**: irreducible polynomial P
- RSA: composite integer $n = pq$

Private key:

- **TCHo**: a sparse multiple of P
- RSA: the prime factors of n

Hard problem:

- **TCHo**: finding a sparse multiple (polynomial)
- RSA: finding a prime factor (integer)

Encryption:

- **TCHo**: probabilistic
- RSA: deterministic

DESCRIPTION OF **TCHo**

ENCRYPTION

10101001 . . . 10101001	repetition of $m\|\|m\|\| \ldots \|\|m$
$\oplus$	
01110110 . . . 01101110	output of a LFSR with random state
$\oplus$	
00100100 . . . 00100010	random bits with bias $\gamma = \Pr(0) - \Pr(1)$

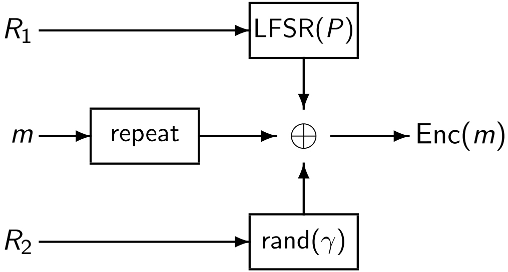
such that

- LFSR feedback polynomial is the public key P
- $\gamma > 0$ (more zeros than ones)
- the ciphertext is a ℓ-bit string, with $\ell \geq \deg(K)$

$$\mathrm{Enc}(m) = m|| \ldots ||m \oplus \mathrm{LFSR}(P) \oplus \mathrm{rand}(\gamma)$$

ENCRYPTION

Implementation is built on three independent components, fed with two random (unbiased) samples R_1 and R_2
$\Rightarrow$ parallelizable



DECRYPTION

$$\begin{array}{ll} K & \text{private key, sparse multiple of } P \\ \otimes & \\ 10011011\ldots10101011 & c = m||\ldots||m \oplus \mathrm{LFSR}(P) \oplus \mathrm{rand}(\gamma) \\ = 0100\ldots1101 & m'||\ldots||m' \oplus \mathrm{rand}(\gamma^{\mathrm{w}(K)}) \end{array}$$

$\Rightarrow$ can compute m' (count majority), and recover m:

$$m \leftarrow \psi(m')$$

ψ is a linear mapping defined by K

PRODUCT POLYNOMIAL$\otimes$BITSTRING

Let $K = \sum k_i x^i$, and a bitstring $u = (u_0, \ldots, u_{\ell-1})$, then $K \otimes u = v$, with v of $\ell - \deg(K)$ bits, and

$$v_i = u_i k_0 + \cdots + u_{i+\deg(K)} k_{\deg(K)}$$

$\approx$ sequence of dot products

Properties exploited in decryption (recall $K = P \times P'$)

- $K \otimes \Big(\text{output of LFSR with feedback } P\Big) = 0 \ldots 0$
- $K \otimes \Big(\text{output} \ldots \oplus \text{rand}(\gamma)\Big) \approx \text{rand}(\gamma^{\text{w}(K)})$

DECRYPTION

K	private key, sparse multiple of P
$\otimes$	
$10011011\ldots10101011$	$c = m\|\|\ldots\|\|m \oplus \mathrm{LFSR}(P) \oplus \mathrm{rand}(\gamma)$
$= 0100\ldots1101$	$m'\|\|\ldots\|\|m' \oplus \mathrm{rand}(\gamma^{\mathrm{w}(K)})$

$\Rightarrow$ can compute m' (count majority), and recover m:

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DECRYPTION RELIABILITY

$\psi(m)$ repeated

$$N = \frac{\ell - \deg(K)}{|m|} \text{ times}$$

Decrypt incorrectly $\Leftrightarrow$ majority logic fails $\Leftrightarrow$ at least one bit of $\psi(m)$ is noised more than half the times.

$$\Pr[\text{bad decryption}] \approx |m| \cdot \varphi\Big(- \sqrt{\frac{N\gamma^{2w}}{1-\gamma^{2w}}}\Big)$$

with φ the cumulative distribution of $\mathcal{N}(0,1)$.

KEY GENERATION

Problem: find a pair (K, P), with K a sparse multiple of P, of given degree and weight, and P of degree in $[d_{\mathrm{min}}, d_{\mathrm{max}}]$.

Until a suitable P is found, repeat

- pick a random K of given degree and weight
- factorize it
- look for an irreducible P of suitable degree in K's factors

(in practice large degrees: $\deg(K) > 15\,000, \deg(P) > 5\,000$)

EXAMPLE OF PARAMETERS

For 80-bit security,

- plaintext of $|m| = 128$ bits
- ciphertext of $\ell = 56\,000$ bits
- public-key is polynomial P of degree $\in [7\,150, 8\,000]$
- private-key is polynomial K of degree $24\,500$ and weight 51
- noise has bias 0.98
- decryption fails with probability 2^{-23}

SECURITY OF **TCHo**

PRIVATE KEY RECOVERY

We can decrypt

- if we recover K, sparse multiple of the polynomial P, OR
- if we find another sparse multiple of degree $\leq \deg(K)$

Computational problem **LWPM**

- Parameters: w, d, d_P, $0 < d_P < d$ and $w \ll d$.
- Instance: P of degree d_P
- Question: find a multiple of P of degree $\leq d$ and weight $\leq w$.

Strategies: exhaustive search, generalized birthday paradox, syndrome decoding.

In **TCHo**, the existence of a solution is guaranteed !

PRIVATE KEY RECOVERY

Computational problem **LWPM**

- Parameters: w, d, d_P, $0 < d_P < d$ and $w \ll d$.
- Instance: P of degree d_P
- Question: find a multiple K of P, s.t. $\deg(K) \leq d$ AND $\mathrm{w}(K) \leq w$.

Strategies: exhaustive search, generalized birthday paradox, syndrome decoding.

In **TCHo**, the existence of a solution is guaranteed !

LWPM requires $\Omega(2^{\lambda})$ operations if

$$\binom{d}{w-1} \leq 2^{d_P} \quad \text{and} \quad w \log \frac{d}{d_P} \geq \lambda$$

BASIC SECURITY PROPERTIES

TCHo is trivially malleable,

$$\mathrm{Enc}(m) \oplus \Delta = \mathrm{Enc}(m \oplus \Delta)$$

TCHo can be inverted by a CCA adversary: given challenge ciphertext c, just query for $m \leftarrow \mathrm{Dec}(c \oplus \Delta)$, and recover original message $m \oplus \Delta$.

TCHo can be used as a KEM in hybrid encryption scheme, to provide IND-CCA security.

SEMANTIC SECURITY

Consider the problem of distinguishing

$$c = \mathrm{LFSR}(P) \oplus \mathrm{rand}(\gamma) \oplus (m || \ldots || m)$$

for a chosen m, from

$$\mathrm{rand}(0)$$

(real-or-random game)

challenge XORed with m gives either

$$\mathrm{LFSR}(P) \oplus \mathrm{rand}(\gamma) \text{ OR } \mathrm{rand}(0)$$

Reduction to **Noisy LFSR**: distinguish (ℓ-bit strings)

- $\mathrm{LFSR}(P) \oplus \mathrm{rand}(\beta)$ from
- $\mathrm{rand}(0)$

SEMANTIC SECURITY

Noisy LFSR: distinguish

- LFSR(P) $\oplus$ rand(β) from
- rand(0)

$P \otimes$challenge$=$ either rand($\gamma^{\mathrm{w}(P)}$) or rand(0).

$\Rightarrow$ **Noisy LFSR** solvable if can distinguish rand($\gamma^{\mathrm{w}(P)}$) from rand(0)

If we know P' such that $\mathrm{w}(PP') < \mathrm{w}(P)$,
$(PP')\otimes$challenge$=$ either rand($\gamma^{\mathrm{w}(PP')}$) or rand(0).

$\Rightarrow$ **Noisy LFSR** solvable if can distinguish rand($\gamma^{\mathrm{w}(PP')}$) from rand(0)

but less bits than with P!

SEMANTIC SECURITY

With the previous method, we get a ratio $\frac{\text{advantage}}{\text{complexity}}$

$$\max_{w \in [0, d_P], N \geq 1} \sqrt{\frac{N}{2\pi}} \frac{\gamma^w}{wN + 2^{\deg(P)} \left(\frac{\ell}{d_P}\right)^{w-1} \binom{\ell}{w}^{-1}}$$

with N the number of bits with bias $\gamma^{\mathrm{w}(PP')}$ used,

Theorem
Assuming the hardness of **LWMP** and **Noisy LFSR**,
TCHo is semantically secure.

PERFORMANCES OF **TCHo**

PERFORMANCES

Recall parameters: $|m| = 128$, $|\mathrm{Enc}(m)| = 56\,000$,
$\deg(P) \in [7\,150, 8\,000]$, $\deg(K) = 24\,500$, $\mathrm{w}(K) = 51$, $\gamma = 0.98$.

Average timings with C++ & NTL, gcc 3, over Intel P4 1.5GHz.
NTL used for matrix inversion and polynomial factorization (Cantor-Zassenhaus).
Biased random bits generated in 2 steps: 1) pick weight k w.r.t. γ, 2) pick word of weight k.

Timings:

- Encryption: 90ms (bottleneck = LFSR output computation)
- Decryption: 65ms (bot. = product ciphertext$\otimes K$)
- Key generation: 30min (bot. = factorization)

(timings include precomputation of ψ)

PERFORMANCES

Flexible parameters (trading-off ciphertext length, key gen. time, enc/dec. time, etc.). For example with parameters $|m| = 128$, $|\mathrm{Enc}(m)| = 150\,000$, $\deg(P) \in [6\,000, 8\,795]$, $\deg(K) = 17\,600$, $\mathrm{w}(K) = 81$, $\gamma = 0.9766$.

- Encryption: 228ms
- Decryption: 424ms
- Key generation: 2min20s

These are software timings, **TCHo** is for hardware!

PERFORMANCES

"Why do you give software timings for a hardware cipher??"
$\rightarrow$ did not have the opportunity to implement HW.

Expected much faster on hardware devices, because of

- efficient LFSR
- only GF(2) linear algebra
- parallelization

but key generation. . .

CONCLUSION

SUMMARY

TCHo is. . .

- based on the hardness of recovering a sparse polynomial multiple
- semantically secure
- post-quantum
- flexible
- fast in hardware (except key gen.)

FURTHER WORK

more experiments. . .

- benchmarks on FPGA, ASIC, etc.
- suitable for passive RFID tags ?

more analysis. . .

- speed-up key generation
- replace huge LFSR by. . . ?
- weak instances ?
- solve **LWPM** efficiently ?
- solve **Noisy LFSR** efficiently ?

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